Exercise 1.64

A copper refinery produces a copper ingot weighing 150 lb. If the copper is drawn into wire whose diameter is 7.50 mm, how many feet of copper can be obtained from the ingot? The density of copper is 8.94 g/cm^3. (Assume that the wire is a cylinder whose volume $V = \pi r^2 h$, where r is its radius and h is its height or length.)

Solution

Solve the volume equation for the length.

$$h = \frac{V}{\pi r^2}$$

The volume is mass divided by density.

$$h = \frac{\left(\frac{m}{\rho}\right)}{\pi r^2}$$

Now plug in the numbers, making sure the units cancel appropriately.

$$h = \frac{m}{\pi r^2 \rho}$$

$$= \frac{150 \text{ lb}}{\pi \left(\frac{7.50 \text{ mm}}{2}\right)^2 \left(8.94 \frac{\text{g}}{\text{cm}^3}\right)}$$

$$= \frac{150 \text{ lb} \times \frac{453.59 \text{ g}}{1 \text{ lb}}}{\pi \left(\frac{7.50 \text{ mm}}{2} \times \frac{1 \text{ cm}}{10 \text{ mm}}\right)^2 \left(8.94 \frac{\text{g}}{\text{cm}^3}\right)}$$

$$= \frac{150 \times 453.59 \text{ g}}{\pi \left(\frac{7.50}{2} \times \frac{1}{10}\right)^2 \times 8.94 \frac{\text{g}}{\text{cm}}}$$

$$= \frac{150 \times 453.59 \text{ cm}}{\pi \left(\frac{7.50}{2} \times \frac{1}{10}\right)^2 \times 8.94} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in}}$$

$$\approx \begin{cases} 5.7 \times 10^2 \text{ ft} & \text{if the uncertainty in 1}{\color{red}5}\text{0 lb is in the tens place} \\[2mm] 5.65 \times 10^2 \text{ ft} & \text{if the uncertainty in 15}{\color{red}0}\text{ lb is in the ones place} \end{cases}$$